[PHYSICS]

1. The acceleration due to gravity at a depth d below surface of the earth is,

$$g' = \frac{GM}{R^2}\left(1 - \frac{d}{R}\right) = g\left(1 - \frac{d}{R}\right)$$

$g' = 0$ at $d = R$

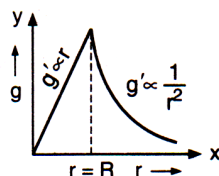*i.e.*, acceleration due to gravity is zero at the centre of the earth.
Thus, the variation in value of g with r is:

For $r > R$:

$$g' = \frac{g}{\left(1 + \dfrac{h}{R}\right)^2} = \frac{gR^2}{r^2}$$

or $\qquad g' \propto \dfrac{1}{r^2}$

Here, $R + h = r$

For $r < R$: $\qquad g' = g\left(1 - \dfrac{d}{R}\right) = \dfrac{gr}{R}$

Here, $\qquad R - d = r$
or $\qquad g' \propto r$

Therefore, the variation of g with distance from centre of the earth will be as shown in the figure above.

2. D

3. Here, angular momentum is conserved, *i.e.*, $L = I\omega =$ Constant. At A, the moment of inertia I is least, so angular speed and therefore the linear speed of planet at A is maximum.

4. For the satellite, the gravitational force provides the necessary centripetal force, *i.e.*,

$$\frac{GM_e m}{(R + x)^2} = \frac{mv_o^2}{(R + x)} \quad \text{and} \quad \frac{GM_e}{R^2} = g$$

$$\therefore \qquad v_o = \sqrt{\frac{gR^2}{R + x}}.$$

5. Potential energy of mass m when it is midway between masses M_1 and M_2

$$U = -\frac{GM_1 m}{d/2} - \frac{GM_2 m}{d/2} = -\frac{2Gm}{d}(M_1 + M_2)$$

According to law of conservation of energy,

$$\frac{1}{2}mv_e^2 = \frac{2Gm}{d}(M_1 + M_2)$$

$$v_e = \text{escape velocity} = \sqrt{\frac{4G(M_1 + M_2)}{d}}.$$

6. Since, orbital velocity is inversely proportional to the square root of the radius of the orbit,
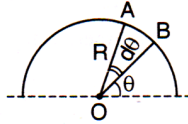
$$\frac{v_A}{v_B} = \sqrt{\frac{R_B + R_e}{R_A + R_e}}.$$

7. **At the point P,** we have $I_1 - I_2 = 0$ (because the gravitational field inside a shell is zero). Hence, $I_1 = I_2$.

8. Mass per unit length of the rod $= \dfrac{M}{L}$

Mass of small length element AB,

$$dm = \left(\frac{M}{L}\right) R\, d\theta$$

Potential at O due to this element,

$$dV = -G \cdot \frac{dm}{R} = -\frac{GM}{L}\frac{R\, d\theta}{R} = -\frac{GM}{L}\, d\theta$$

$$\therefore \qquad V = \int dV = -\frac{GM}{L}\int_0^\pi d\theta = -\frac{\pi GM}{L}.$$

9. $$V_S = -\frac{GM}{R} = V$$

$$V_C = -\frac{GM}{2R^3}(3R^2 - r^2)$$

$$= -\frac{GM}{2R^3}(3R^2 - 0) = -\frac{3}{2}\frac{GM}{R} = \frac{3}{2}V.$$

10. $$\omega = \frac{2\pi}{24 \times 3600}$$

$$g' = g - R_e \omega^2 \quad \text{or} \quad 0 = g - R_e\, \omega'^2$$

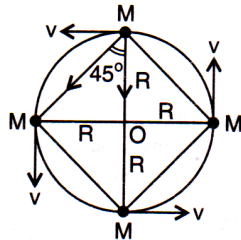$$\text{or} \qquad \omega' = \sqrt{\frac{g}{R_e}} = \frac{2\pi}{84.6 \times 60}$$

$$\therefore \qquad \frac{\omega'}{\omega} = \frac{24 \times 3600}{84.6 \times 60} = 17.$$

11. Gravitational force on each due to other three particles provides the necessary centripetal force.

$$\sqrt{2}\,\frac{GM^2}{(\sqrt{2}R)^2}\cos 45° + \frac{GM^2}{(2R)^2} = \frac{Mv^2}{R}$$

Simplifying it, we get;

$$v = \sqrt{\frac{GM}{R}\left(\frac{2\sqrt{2}+1}{4}\right)}.$$

12. C

13. B

14. By the principle of superposition of fields,

$$\vec{E} = \vec{E_1} + \vec{E_2}$$

Here, $\vec{E}$ = net field at the centre of hole due to entire mass

$\vec{E_1}$ = field due to remaining mass

and $\vec{E_2}$ = field due to mass in hole = 0

$\therefore \qquad \vec{E_1} = \vec{E} = \dfrac{GM}{R^3} r \qquad \left(\text{where } r = \dfrac{R}{2} \right)$

$\therefore \qquad \vec{E} = \dfrac{GM}{2R^2} .$

15. We know from the Kepler's law that the line between a planet and the sun sweeps out equal areas in equal times.

16. For a satellite revolving round the earth in circular orbit of radius r,

$$\dfrac{mv_o^2}{r} = \dfrac{GMm}{r^2} \quad \text{or} \quad v_o = \sqrt{\dfrac{GM}{r}}$$

17. $\qquad \dfrac{GMm}{R^2} = m\omega^2 R$

$\therefore \qquad KE = \dfrac{1}{2} I\omega^2 = \dfrac{1}{2} mR^2\omega^2 = \dfrac{GMm}{2R}$

$\qquad PE = -\dfrac{GMm}{R}$

$\therefore \qquad KE = \dfrac{|PE|}{2}$

or $\qquad \dfrac{KE}{|PE|} = \dfrac{1}{2} .$

18. $\qquad T_1 = 2\pi\sqrt{\dfrac{l}{g}}; \quad T_2 = 2\pi\sqrt{\dfrac{l}{g'}}$

$\qquad g' = g \dfrac{R^2}{(R+h)^2} = \dfrac{gR^2}{(R+2R)^2} \qquad (\because h = 2R)$

$\qquad g' = \dfrac{g}{9}$

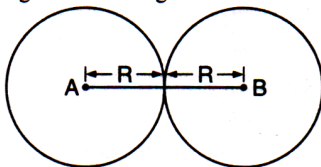$\qquad T_2 = 2\pi\sqrt{\dfrac{l}{g/9}}$

$\qquad = 2\pi\sqrt{\dfrac{9l}{3}} = 3T_1$

or $\qquad \dfrac{T_1}{T_2} = \dfrac{1}{3} .$

19. Let masses of two balls are $m_1 = m_2 = m$ (given) and the density be ρ.

Distance between their centres = $AB = 2R$

Thus, the magnitude of the gravitational force F that two balls

20.

21. Kinetic energy of a satellite, $E = \dfrac{1}{2}mv^2$

where m = mass of a satellite and v = orbital velocity of a satellite

$E = \dfrac{1}{2}mv^2$ or $mE = \dfrac{1}{2}m^2v^2$ or $mv = \sqrt{2mE}$

Angular momentum, $L = mvr = \sqrt{2mE}\,r = \sqrt{2mEr^2}$.

22.

23.

24. Change in PE, $\Delta U = U_2 - U_1$

$\therefore \qquad \Delta U = -\dfrac{GMm}{(R + nR)} + \dfrac{GMm}{R}$

or $\qquad \Delta U = -\dfrac{GMm}{R(1 + n)} + \dfrac{GMm}{R}$

$\qquad\qquad = \dfrac{GMm}{R}\left[-\dfrac{1}{1 + n} + 1\right]$

or $\qquad \Delta U = \dfrac{(R^2 g)m}{R} \times \dfrac{n}{1 + n} \qquad \left[\because g = \dfrac{GM}{R^2}\right]$

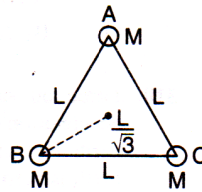or $\qquad \Delta U = mgR\left(\dfrac{n}{1 + n}\right)$.

25. Given $F_1 = F_2 = F$ and $\theta = 60°$

Resultant force = $\sqrt{3}\,F$

$\therefore$ Force on mass at A due to mass at B and C

$\qquad = \sqrt{3}\left(\dfrac{GM^2}{L^2}\right)$

Centripetal force for circumscribing the triangle in a circular orbit is provided by mutual gravitational interaction.

$i.e., \qquad \dfrac{Mv^2}{(L/\sqrt{3})} = \sqrt{3}\left(\dfrac{GM^2}{L^2}\right)$

$\qquad\qquad v = \sqrt{\dfrac{GM}{L}}$

[CHEMISTRY]

46. (c) : Rate $= -\dfrac{d[N_2O_5]}{dt} = +\dfrac{1}{2}\dfrac{d[NO_2]}{dt}$

$$= 2\dfrac{d[O_2]}{dt}$$

Given $\dfrac{-d[N_2O_5]}{dt} = 6{\cdot}25 \times 10^{-3}$ mol L^{-1} s^{-1}

Rate of formation of NO$_2$

$$= \dfrac{[NO_2]}{dt} = -2\dfrac{d[N_2O_5]}{dt}$$

$$= 2 \times 6{\cdot}25 \times 10^{-3} \text{ mol L}^{-1}\text{ s}^{-1}$$

$$= 12{\cdot}50 \times 10^{-3} \text{ mol L}^{-1}\text{ s}^{-1}$$

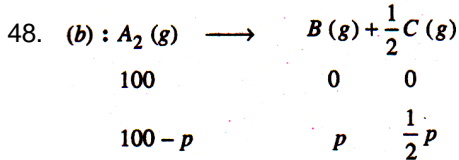$$= 1{\cdot}25 \times 10^{-2} \text{ mol L}^{-1}\text{ s}^{-1}$$

Rate of formation of O$_2$

$$= \dfrac{d[O_2]}{dt} = -\dfrac{1}{2}\dfrac{d[N_2O_5]}{dt}$$

$$= \dfrac{1}{2} \times 6{\cdot}25 \times 10^{-3} \text{ mol L}^{-1}\text{ s}^{-1}$$

$$= 3{\cdot}125 \times 10^{-3} \text{ mol L}^{-1}\text{ s}^{-1}$$

47. (b) : Minus signs are for reactants and positive signs for products. Dividing numbers are the coefficients.

48. (b) :

$$A_2\,(g) \longrightarrow B\,(g) + \dfrac{1}{2}C\,(g)$$

100	0	0
$100 - p$	p	$\dfrac{1}{2}p$

$$100 - p + p + \dfrac{1}{2}p = 120 \text{ or } p = 40 \text{ mm}$$

$$\therefore\; -\dfrac{dp_{A_2}}{dt} = \dfrac{40}{5} = 8 \text{ mm min}^{-1}$$

49. (c) : Initially, Rate $= k\,[Y]\,[Z]^{1/2}$

New rate $= k\,[Y]\,[2\,Z]^{1/2}$

$$= \sqrt{2}\,k\,[Y][Z]^{1/2} = 1{\cdot}414\,k\,[Y]\,[Z]^{1/2}.$$

50. (d) : The rate of reaction is same as expressed in terms of any reactant or product.

51. (a) : Rate of reaction $= \dfrac{1}{4}\dfrac{\Delta[NO_2]}{\Delta t}$

$$= \dfrac{1}{4} \times \dfrac{[5{\cdot}2 \times 10^{-3}\,M]}{100\,s} = 1{\cdot}3 \times 10^{-5} \text{ M } s^{-1}.$$

52. (b) : Rate $= k\,[NO]^2\,[O_2]$. Initially rate $= ka^2 b$. If volume is reduced to half, concentration are doubled so that new rate

$$= k\,(2\,a)^2\,(2\,b) = 8\,k\,a^2\,b,\ i.e.,\ 8 \text{ times.}$$

53. (c) : From slow step, rate = k [B_2] [A].

From 1st eqn, $K_{eq} = \dfrac{[A]^2}{[A_2]}$

or $\quad [A] = \sqrt{K_{eq}[A_2]} = K_{eq}^{1/2}[A_2]^{1/2}$

Hence, rate $= k\,[B_2]\,K_{eq}^{1/2}\,[A_2]^{1/2}$

$\qquad\qquad = k'\,[A_2]^{1/2}\,[B_2]$.

Hence, order $= 1\dfrac{1}{2}$.

54. (c) : On the basis of given units of k, the reaction is of 3rd order.

55. (d) : $r = k\,[A]^\alpha\,[B]^\beta = k\,a^\alpha\,b^\beta$. If concentration of B is doubled, $\dfrac{r}{4} = k\,a^\alpha\,(2\,b)^\beta$. Dividing 2nd eqn. by 1st eqn.,

$\dfrac{1}{4} = 2^\beta$ or $2^\beta = 2^{-2}$. Hence, $\beta = -2$.

56. (a) : As step I is the slowest, hence it is the rate determining step.

57.

58. (d) : $k = \dfrac{2.303}{32}\log\dfrac{a}{a - 0.99\,a}$

$\qquad = \dfrac{2.303}{32}\log 10^2 = \dfrac{2.303}{16}\ \text{min}^{-1}$

$t_{99.9\%} = \dfrac{2.303}{k}\log\dfrac{a}{a - 0.999\,a}$

$\qquad = \dfrac{2.303}{k}\log 10^3 = \dfrac{3 \times 2.303}{2.303} \times 16$

$\qquad = 48$ min.

59. (b) : 0.08 mol L^{-1} to 0.01 mol L^{-1} involves 3 half-lives.

60. (d) : $k = \dfrac{2.303}{t}\log\dfrac{a}{a - x}$

or $\log\dfrac{a}{a - x} = \dfrac{kt}{2.303} = \dfrac{2.2 \times 10^{-5} \times 60 \times 90}{2.303}$

$\qquad\qquad\qquad = 0.0516$.

Hence, $\dfrac{a}{a - x}$ = antilog $0.0516 = 1.127$.

or $\quad \dfrac{a - x}{a} = 0.887$ or $1 - \dfrac{x}{a} = 0.887$

or $\dfrac{x}{a} = 0.113 = 11.3\%$.

61. (c) : $t_{90\%} = \dfrac{2.303}{k} \log \dfrac{a}{a - 0.9a}$

$= \dfrac{2.303}{k} \log 10 = \dfrac{2.303}{k}$

$t_{1/2} = \dfrac{2.303}{k} \log \dfrac{2}{a - a/2}$

$= \dfrac{2.303}{k} \log 2 = \dfrac{2.303}{k} \times 0.3010$

$\therefore t_{90\%}/t_{1/2} = \dfrac{1}{0.3010} = 3.3$

i.e., $t_{90\%} = 3.3$ times $t_{1/2}$.

62. (a) : Decrease in concentration from 0.8 M to 0.4 M in 15 minutes means $t_{1/2}$ = 15 minutes. Time taken for decrease in concentration from 0.1 M to 0.25 M means two half-lives, i.e., = 2 × 15 min = 30 min.

63. (c) : At the point of intersection, [A] = [B], i.e., half of the reactant has reacted. Hence, it represents $t_{1/2}$.

64 (a) : It P_t is the pressure after time t,

$$k = \dfrac{2.303}{t} \log \dfrac{P_0}{P_t}$$

$\therefore 3.38 \times 10^{-5} \, s^{-1} = \dfrac{2.303}{600 \, s} \log \dfrac{500 \text{ atm}}{P_t}$

or $\log \dfrac{500}{P_t} = 0.0088$ or $\dfrac{500}{P_t} = 1.021$

or $P_t = 490$ atm

65. (a) : [A] is kept constant, [B] is doubled, rate is doubled. So rate $\propto$ [B].
[B] is kept constant, [A] is tripled, rate becomes 9 times, so rate $\propto [A]^2$.
Hence, rate law is
rate = $k [A]^2 [B]$

66 (c) : The reaction occurring in two steps has two activation energy peaks.
The first step, being fast needs less activation energy. The second step, being slow, needs more activation energy. Therefore, second peak will be higher than the first.

67. (b) : Rate = k [NOBr$_2$] [NO]

From step I, $K_{eq} = \dfrac{[NOBr_2]}{[NO] [Br_3]}$

or [NOBr$_2$] = K_{eq} [NO] [Br$_2$]
Substituting in eqn. (i), we get
Rate = $k K_{eq}$ [NO]^2 [Br$_2$] = k' [NO]^2 [Br$_2$]
Hence, order with respect to NO is 2.

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68. (b) : $r = K [A]^\alpha = k\,a^\alpha$

$1\cdot837\,r = k\,(1\cdot5\,a)^\alpha$

Dividing, $1\cdot837 = (1\cdot5)^\alpha$

On solving, we get $\alpha = 1\cdot5$

Hence order = $1\cdot5$

69. (a) : Half-life of a frist order reaction does not depend upon initial concentration. It is equal to ln $2/k$.